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LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2014**

MT 5509/MT 5508/MT 5502 - ALGEBRAIC STRUCTURE - II

Date : 05/11/2014 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

PART – A

Answer ALL questions: **(10 x 2 = 20)**

1. Define a vector space over a field F.
2. Give an example to show that the union of two subspaces of a vector space V need not be a subspace of V.
3. Prove that any $n + 1$ vectors in F_n are linearly independent.
4. Define Kernel and Image of a homomorphism T.
5. Define an algebra over a field F.
6. What is eigen value and eigen vector?
7. Define trace of a matrix A.
8. When do you say that a matrix is invertible?
9. When is a linear transformation T on V said to be unitary?
10. Define rank of a matrix.

PART – B

Answer any FIVE questions: **(5 x 8 = 40)**

11. Show that the set of all m x n matrices over F is a vector space under matrix addition and scalar multiplication of a matrix.
12. Prove that the intersection of two subspaces of a vector space V is a subspace of V.
13. If V is a vector space of finite dimension and W is a subspace of V, then prove that dim V / W = dim V – dim W.
14. Prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is non-zero.
15. Prove that $T \in A(V)$ is invertible if and only if whenever $v_1, v_2, \ldots v_n$ are in V and linearly independent, then $T(v_1), T(v_2), \ldots T(v_n)$ are also linearly independent.
16. Show that any square matrix can be expressed uniquely as the sum of a symmetric and skew – symmetric matrices.
17. If V has dimension n and $T \in A(V)$ then prove that the rank of T is equal to the rank of the corresponding matrix $m(T)$ in F_n.
18. Show that the system of equation $x + 2y + z = 11,\ 4x + 6y + 5z = 8\ \&\ 2x + 2y + 3z = 19$ is inconsistent.

PART – C

Answer any TWO questions: (2 x 20 = 40)

19. Let V be a vector space over a field F and W be a subspace of V. Prove that V/W is a vector space over F.

20. a) If S,T$\in$A(V) then prove the following:
    (i) rank (ST) $\leq$ rank of T
    (ii) rank (ST) $\leq$ rank of S
    (iii) rank (ST) = rank of (TS) = rank of T for an invertible S in A(V).

    b) If V is a vector space over F, then define Hom (V, V) and prove that Hom(V,V) is an algebra ever the field F.

21. Prove that every finite dimensional inner product space V has a orthonormal set as a basis.

22. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & -1 \\ 0 & -1 & -2 \end{pmatrix}$.

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